Università Milano-Bicocca

Fixed Point Techniques in Economics.

Reading group.

Ph.D. in Economics – Università degli Studi Milano-Bicocca
Ph.D. in Statistics – Università Cattolica Milano

Academic Year 2020-21

Instructor: Gianluca Cassese (gianluca.cassese@unimib.it)

Dates

The meetings will take place on the following dates:

Friday	15/01	16:30	17:30
Tuesday	19/01	16:30	18:30
Friday	22/01	16:30	18:30
Tuesday	26/01	16:30	18:30
Friday	29/01	16:30	18:30
Tuesday	02/02	16:30	18:30
Friday	05/02	16:30	18:30
Tuesday	09/02	16:30	18:30

Students should connect remotely by accessing the website

https://unimib.webex.com/meet/gianluca.cassese

Course description

The aim of this reading group is to introduce students to the mathematical techniques of general equilibrium and of game theory that are related to fixed point theorems. We will get into the most useful and used results of fixed point theory, stating and proving some of the most important results of this literature. At the same time we shall try as much as possible to provide examples and suggest applications.

My lectures will cover approximately 12 hours. Students will be asked to study one or two papers and to offer a detailed presentation of these. The papers should be chosen from the appended list but may also be selected from other sources, with the consent of the instructor.

Prerequisites

The course will be highly mathematical and there will be but very short time to review the basic definitions of a metric, a topological and a Banach space as well as other main mathematical concepts. It is advisable that students have previously been exposed to such notions or at least that they be willing to catch up on their own.

Objectives

The purpose is to revise these technical results in order to permit students to approach general equilibrium and game theory literature with relative ease. In particular, we shall go through the following, important results:

(1) Banach contraction mapping;
(2) Knaster, Kuratowski and Mazurkiewicz Lemmas;
(3) Fixed point theorems of (*a*) Brower, (*b*) Browder (*c*) Fan-Glicksberg and (*d*) Kakutani.
(4) continuity of correspondences
(5) budget sets and excess demand correspondences;
(6) the general equilibrium theorem of Arrow and Debreu;
(7) other approaches to general equilibrium.

Assessment

Student are required to deliver a presentation of approx. 30 minutes based on the reading of one or more papers. The talk will be followed by a short discussion. The presenter should be able to expose the paper with clarity and to be able to reply to questions. The evaluation is based on the effectiveness of the presentation. Attendance of the course is mandatory and active participation is sought for.

Textbooks and reference material

A tentative list of references includes:

(1) Arrow, K. J., and Debreu, G. *Existence of an equilibrium for a competitive economy. Econometrica 22*, 4 (1954), 265–290.

(2) Border, K. *Fixed Point Theorems with Applications to Economics and Game Theory*. Cambridge University Press, Cambridge, 1985.

(3) Debreu, G. *Theory of value. An axiomatic analysis of economic equilibrium.* Cowles foundation monograph N. 17. Yale University Press, New Haven, 1970.

(4) Debreu, G. Chapter 15 existence of competitive equilibrium. vol. 2 of *Handbook of Mathematical Economics*. Elsevier, 1982, pp. 697 – 743.

(5) Florenzano, M. *General equilibrium analysis. Existence and optimality properties of Equilibria*. Springer, Dordrecht, 2003.

(6) Geanakoplos, J. *Arrow-Debreu Model of General Equilibrium*. Palgrave Macmillan UK, London, 2016, pp. 1–15.

(7) Mas-Colell, A., Whinston, M. D., and Green, J. *Microeconomic Theory*. Oxford University Press, Oxford, 2018.

(8) Mas-Colell, A., and Zame, W. *Equilibrium theory in infinite dimensional spaces*. In *Handbook of Mathematical Economics*, W. Hildenbrand and H. Sonnenschein, Eds., 1 ed., vol. 4. Elsevier, 1991, ch. 34, pp. 1835–1898.

A list of papers suggested for presentation will be available as soon as possible.